4/19/18

"You can try and take the high road, but you need to know where it is." -Unknown

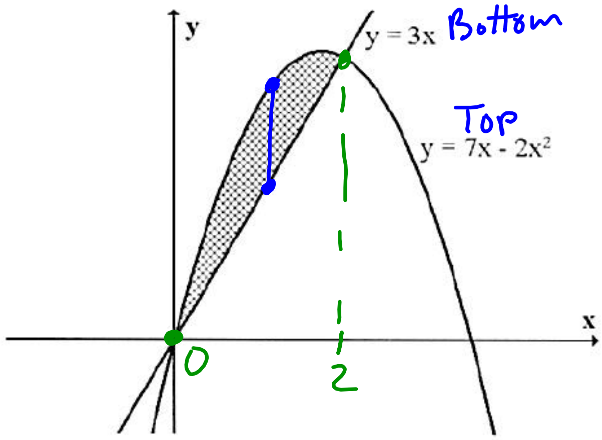
HW: "2017 Calc L32 Area Between Curves" #5-7

Test 1 on Wednesday 5/2

AIM: More Area Between Curves

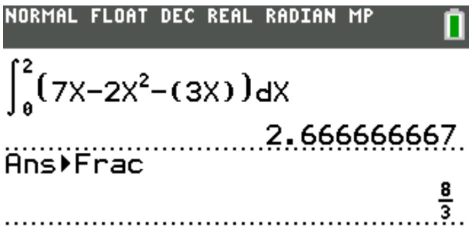
3. The diagram opposite shows the curve $y = 7x - 2x^2$ and the line $y = 3x$.

Calculate the shaded area.



$$3x = 7x - 2x^2$$
$$-7x + 2x^2 \quad -7x + 2x^2$$
$$2x^2 - 4x = 0$$
$$2x(x-2) = 0$$
$$x = 0 \quad x = 2$$

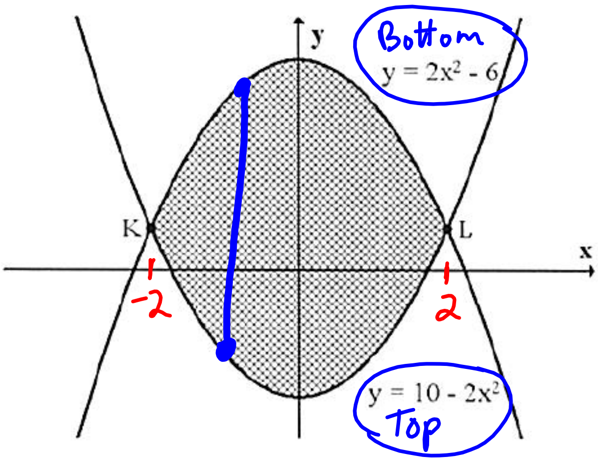
$$\text{Area} = \int_0^2 \left(7x - 2x^2 - (3x)\right) dx$$



$$\boxed{\text{Area} = \frac{8}{3} \text{ units}^2}$$

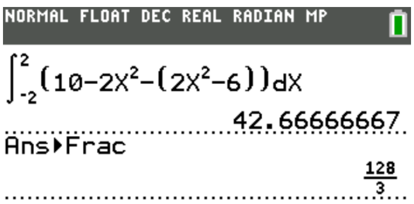
4. The curves with equations $y = 2x^2 - 6$ and $y = 10 - 2x^2$ intersect at K and L.

Calculate the area enclosed by these two curves.



$$10-2x^2 = 2x^2-6$$
$$-10+2x^2 \quad +2x^2-10$$
$$0 = 4x^2-16$$
$$0 = 4(x^2-4)$$
$$0 = 4(x+2)(x-2)$$
$$x=-2 \quad x=2$$

$$\text{Area} = \int_{-2}^{2} \left(10-2x^2-(2x^2-6)\right)dx$$

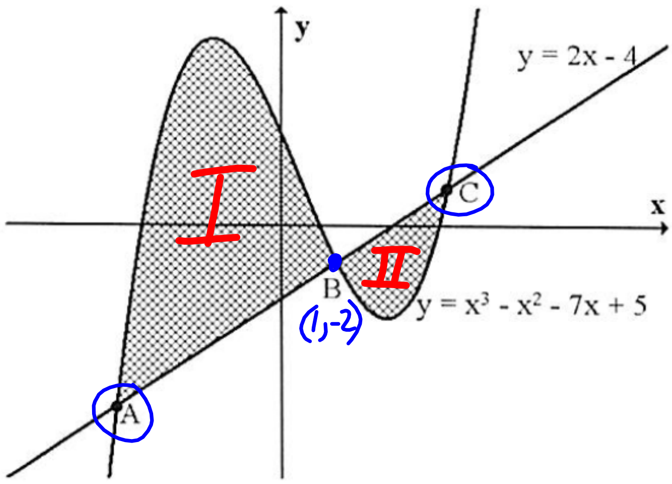


$$\boxed{\text{Area} = \frac{128}{3} \text{ units}^2}$$

9. The curve $y = x^3 - x^2 - 7x + 5$ and the line $y = 2x - 4$ are shown opposite.

(a) B has coordinates (1,-2). Find the coordinates of A and C.

(b) Hence calculate the shaded area.



a) $2x - 4 = x^3 - x^2 - 7x + 5$

$0 = x^3 - x^2 - 9x + 9$

$0 = x^2(x-1) - 9(x-1)$

$(x^2 - 9)(x-1)$

$(x+3)(x-3)(x-1)$

$x = -3 \quad x = 3 \quad x = 1$

A = $y = 2(-3) - 4$
$y = -10$

$\boxed{A = (-3, -10)}$

C $y = 2(3) - 4$
$y = +2$

$\boxed{C = (3, 2)}$

b) $\text{Area I} = \int_{-3}^{1} \left(x^3 - x^2 - 7x + 5 - (2x - 4)\right) dx = \frac{128}{3}$

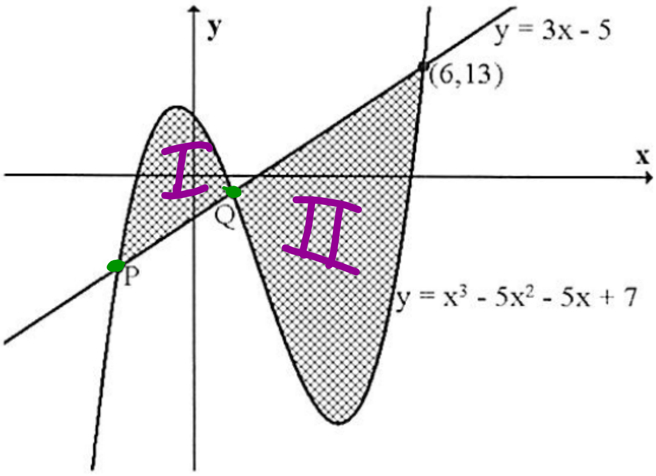
+

$\text{Area II} = \int_{1}^{3} \left(2x - 4 - (x^3 - x^2 - 7x + 5)\right) dx = \frac{20}{3}$

$\text{Total Area} = \frac{128}{3} + \frac{20}{3} = \boxed{\frac{148}{3} \text{ units}^2}$

10. The diagram shows the line $y = 3x - 5$ and the curve $y = x^3 - 5x^2 - 5x + 7$.

(a) Find the coordinates of P and Q.
(b) Calculate the shaded area.



a) $P = (-2, -11)$

$Q = (1, -2)$

b) $\text{Area}_{I} = \int_{-2}^{1} \left(x^3 - 5x^2 - 5x + 7 - (3x-5)\right) dx = \frac{117}{4}$

$\text{Area}_{II} = \int_{1}^{6} \left(3x - 5 - (x^3 - 5x^2 - 5x + 7)\right) dx = \frac{1375}{12}$

Total Area $= \frac{117}{4} + \frac{1375}{12} = \boxed{\frac{863}{6} \text{ units}^2}$